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Construction of Optimal Portfolio Using Sharpe's Single Index Model: An Empirical Study on Nifty 50 Stocks

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Abstract

The construction of an optimal portfolio has become increasingly challenging in recent years, as investors expect to maximize returns and minimize risks from their respective investments. An investor needs to have proper knowledge of security analysis and portfolio theory for making correct investment decisions. In early 1950, Harry Markowitz developed a comprehensive model which stated that investors can reduce their risk through diversification In the present study Sharpe's Single Index Model (SIM) is used to construct an optimal portfolio. The reason for choosing SIM over the Markowitz Model is that it requires fewer inputs and is easier to calculate. It is named as Single Index Model as it uses only a single index for portfolio construction. Further, the proportion of investment of each stock included in the optimal portfolio was also computed.

Keywords: Sharpe's Single Index Model, Optimal Portfolio, Cut off Rate, Systematic Risk, Unsystematic Risk, Diversification, Beta, Risk, Return and Variance

Introduction

The employment of funds on assets with an aim to earn return and capital appreciation is called investment. There is risk and return involved with every investment. An investor needs to have proper knowledge about security analysis and portfolio theory for making correct investment decisions. This can be done either through traditional or modern approach of portfolio construction. A rational investor wants to maximize his return by minimizing his risk. There are several models having the opinion that, by holding assets from different industries in a portfolio, risk can be considerably reduced. Analysts are of the opinion that when at least 15 or more stocks are added to a portfolio, unsystematic risk can considerably be reduced . Studies have shown that systematic risk accounts for about one fourth of the total risk (Mandal, 2013). In the year 1950, Harry Markowitz proposed a new model stating that in order to obtain benefits of diversification, majority of investors invest in a number of securities.He emphasized on the fact that diversification helps to reduce portfolio risk effectively. Markowitz Model is very information intensive and lacks simplicity. As new securities are added in a portfolio, there is greater requirement of computation of co-variance. In order to overcome complexities of Markowitz Model, William Sharpe developed a new and simpler model to analyze the portfolio. According to his model, security's return is correlated to a single index which is usually a market index. All securities that are traded on the exchange will be a part of the market index. The index movement will also indicate the changes in the security prices. Sharpe's Single Index Model (SIM) demands only (3n+2) bits of information whereas it is (n (n+3)/ 2) bits of information in Markowitz model. In SIM, optimal portfolio is constructed by analyzing the reason behind inclusion of a particular stock based on the associated variables under consideration. The present paper seeks to construct an optimal portfolio using Sharpe's Single Index model and taking into consideration stocks of Nifty 50.The study is relevant in present times because the composition of stocks included in Nifty 50 has changed in recent years and not many authors have constructed optimal portfolio during the period considered for study. This study gains more importance as stocks included in Nifty 50 represent majority of market capitalisation of NSE.With the boom in stock market operations post economic reforms and increasing investment in securities in recent years , it would be of great help to present and prospective investors for constructing optimum portfolio and diversify their risk effectively.

Objectives of the Study

1. To examine the concept of SIM empirically.
2. To construct an optimal portfolio using stocks listed in NIFTY 50.
3. To calculate the risk and return of all stocks included in NIFTY 50 using SIM.
4. To calculate respective proportion for each selected stock to be invested in the portfolio.

Literature Review

Chauhan (2014) constructed a portfolio using top ten stocks of NIFTY. In the study, it was found to be an easy and simple method to calculate optimal portfolio. In this method, fewer number of variables are used as compared to Markowitz Model. It is named Single Index Model as it uses only a single index for portfolio construction. Nalini (2014) in her study considered 15 stocks of various sectors from S&P index, taking BSE

Sensex as market index. It was found that risk can be reduced by diversifying the portfolio. Only four stocks were selected in the optimal portfolio.The capital market is still in a developing stage in India, so this model can be of great use. FIIs are investing vastly through stock exchanges**.** Gopalakrishnan (2014) in his study of 13 actively traded scrips of NSE IT Index, concluded that there exists a significant relationship between IT index and S&P and Nifty. It was found that most of the IT stocks were undervalued compared to the current market levels and investors could consider these stocks for inclusion in their portfolios. Vardarajan, (n.d.) in his study of 20 stocks from Steel, Banking and Media and Entertainment sector concluded that SIM acts as a tool for portfolio selection and helps investors in taking informed decisions. The stock market is volatile and investors need to regularly monitor and update their portfolio. Taneja and Bansal **(**2011) found that SIM successfully diversifies almost all unsystematic risk. According to Jensen (1969) portfolio performance can be measured by finding the difference between actual and expected returns of a portfolio at a given point of time, taking into account expected returns, risk less rate of return, systematic risk and return on market portfolio. Investors are generally risk averse and they prefer certain rather than uncertain incomes. If they are holding a stock for a longer period or taking additional risks, they want to be compensated through extra return over and above the risk free rate of return, which is the risk premium.

A natural relationship exists between measures of portfolio performance and efficiency. Alexander and Resnick (1985) stated that the construction of optimal portfolio is based on the composition and location of efficient frontier. To determine these, investors need to find estimates of variance, co variance and expected returns, co variances and variances must be found by the investor. They further conclude that in the presence of a risk free asset, the efficient portfolio will consist of a combination of both, risk free asset and tangency portfolio. Edwin, Martin and Manfred (1978) believe that under alternative sets of assumptions about variance and co variance, common stock returns the optimal portfolio can be easily devised by simple ranking. The advantages of using simple ranking are that portfolio managers can easily understand the unique characteristics of stock, which are to be included in the portfolio and the optimal portfolio can be very easily devised without much efforts. Naveen (1997) constructed an optimal portfolio using stocks of 30 blue-chip companies listed in BSE. It was found in his study that unsystematic risk or residual variance plays a significant role in determining the proportion of investment in optimal portfolio. He further emphasized that portfolio must be evaluated periodically owing to the volatile nature of stock markets.

Research Methodology

Secondary data is used for the study. Data has been collected from websites like (www.nseindia.com) and (www.yahoofinance.com) For the current study, Nifty 50 index is considered as Market Index. Weekly NSE indices and prices of all the 50 stocks of Nifty 50 are taken for the period between August 1, 2010 to July 31, 2015 for computing weekly return of each security as well as market return. The weekly adjusted closing price of individual stocks are considered.The weekly return of stocks is calculated using the formula

$Ri= (R_2-R_1)/R_1*100$

where R_i=return of stock

R_2= adjusted closing price of week 2

R_1=adjusted closing price of week 1

The mean return of individual stock was found by averaging the weekly returns. The average return was calculated using Ms-Excel. The variance of individual stocks was found by using Ms-Excel. The annualized mean return of each stock and market index was calculated. For risk free rate of return, last five years weekly data of MIBOR has been averaged. The beta measuring systematic risk of each stock was calculated. Taking the annualized mean of the stock and market return and beta coefficient, unique cut off rate is calculated. Securities with their 'excess return to beta ratio' either greater or equal to cut off rate are considered for inclusion in optimal portfolio. Thereafter, each selected stock's proportion in optimal portfolio is ascertained, taking into consideration the risk free rate, cut off rate , 'excess return to beta ratio' and unsystematic risk. The data for the study has been supplemented with references from books,journals, periodicals, books and other relevant sources. A number of statistical and financial tools and techniques have been used for analyzing the data.

Conceptual framework of Sharpe's Index Model: Unlike Markowitz's model, this simplified model states that, by comparing return of individual securities with a single index like 'Market Index', relationship existing between each pair of securities can be determined indirectly. The requirements of large data inputs and tedious calculations requirements in Markowitz model is largely reduced (Mandal, 2003). SIM needs only (3n+2) bits of information or simply each security's Alpha and Beta estimates. For SIM, variance of market index, each security's expected return and unsystematic risk also needs to be assessed.It has become more popular as compared to Markowitz Model due to its simplicity.

Assumptions underlying SIM

1. All investors have homogeneous expectations.
2. A uniform holding period is taken into consideration for determining the risk and return of each security.
3. The price movements of securities are greatly influenced by prevailing economic and business conditions.

4. The indices, to which the securities return are correlated, are some securities markets proxy.

Symbols and notations used:

R_i = return of individual security
R_m = return of market index
R_f = risk free rate of return

Risk or variance of a security comprises of two components :a) systematic risk b) unsystematic risk. Variance due to index is the systematic risk and the residual variance is unsystematic risk.

Systematic risk=βi2*variance of market index
= βi2σ2m
Unsystematic risk = σ^2ei
=Total variance-systematic risk
Total risk = Systematic risk + unsystematic risk
Portfolio variance is = σ ^2p
The variance of the return of the security is
σ^2i= βi ^2σ^2m + σ^2ei

Advantages of SIM: The main benefit of SIM is that it simplifies portfolio construction model and has fewer data input requirements as compared to Markowitz. It requires (3n+2) data requirements as compared to n(n+3)/ 2 data inputs of Markowitz. It provides an estimate of security's return as well return of index. It is an important and easy to use method for calculating optimal portfolio (Chauhan, 2014). Sharpe simplified the numerous and complicated calculations for optimal portfolio construction as required in Markowitz model (Nalini, 2014). As compared to its other portfolio counterparts. Sharpe model incurs lesser daily capital charges as per Basel Capital Accord penalties (McAleer and Veiga ,2008).

Limitations

The risk and uncertainty in the market tends to change with time. SIM provides optimal portfolio based on a single point of time only without considering the dynamic nature of the market.. In this model it is assumed that prices of stock move together only because of movement with the market. Many researchers have identified that there are other influences beyond these factors which affect the movement of stocks.

Construction of Optimal Portfolio: The 'excess return to beta ratio' (R_i-R_f/β) decides inclusion of any stock in the optimal portfolio. The reason behind the inclusion of any stock in the optimal portfolio is clearly explained by this ratio (Fischer and Jordan, 1995). The stocks are ranked from highest to lowest on the basis of excess return to beta ratio. A unique cut off rate is calculated and all the stocks whose (R_i-Rf/β) is above the cut off rate are accepted for inclusion in the optimal portfolio. This single value explains the desirability why a stock is included in the optimal portfolio (Fischer and Jordan, 1995)

Steps involved in construction of Optimal Portfolio.

1. The weekly return of individual stocks is calculated using the formula

Ri=(R_2-R_1/$R_{1)}$*100
Where Ri= return of individual stock
R_1=adjusted closing price of week 1
R_2=adjusted closing price of week 2
The weekly returns are then annualized
Using similar formula, the return of market index (R_m) is calculated. This return is annualized to arrive at the annual return of market index.

2. The 'excess return to beta ratio' is calculated for each stock included in the study.
3. The stocks are ranked in descending order as per the beta ratio(Ri-Rf/β).
4. Unsystematic risk = σ^2ei
=Total variance-systematic risk
σ^2ei= σ ^2p- βi2σ2m
5. A cut off rate (Ci) is calculated for all the securities using the formula

$$Ci=\frac{\sigma^2m\sum(Ri-Rf)\beta/\sigma^ei}{1+\sigma^2m\sum \beta i^2/\sigma^ei}$$

6. The proportion of each stock to be invested in portfolio is calculated using the formula

Wi=Zi/∑Zi
Where Zi= βi^2/σ^2ei (Ri-Rf/β-C)

Analysis of Data: Various statistical and financial tools and techniques have been used for optimal portfolio construction using SIM. The data relating to weekly stock prices along with weekly market indices for the period August 1, 2010 to July 31, 2015 have been collected from the website of NSE (www.nseindia.com) and also from (www.yahoofinance.com). Various statistical data such as mean stock return (R_i), mean index return (R_m), variance of individual stock (σ^2i) and market return, beta, systematic risk and unsystematic risk of all the 50 securities have been collected.

Data inputs which have been collected are arranged in Table 1. It is found from Table 1 that stocks like BHEL, Cairn, Gail, Hindalco, NTPC, ONGC, Reliance Industries, Tata Power, Tata Steel, and Vedanta have negative returns. The negative returns may be due to macroeconomic events taking place in the economy. Most of the banks listed in Index have Beta either equal to or more than 1 .Banks like ICICI, IndusInd Bank, PNB and SBI have Beta more than 1. Bank of Baroda has the highest beta of 2.17. Though these stocks with beta greater than 1 are aggressive securities but stock of Bank of Baroda is most volatile of all the stocks. The returns of these banks are more volatile to changes in the market.Adani Ports, HDFC Bank, Tata Power and Hindustan Unilever having beta of 1 show that their returns move along with the market. Some securities like Bharti Airtel (.681), Cairn (.671), Coal India (.71) and Hero Motocorp (.560) have beta less than 1. It can be seen that majority of securities have beta less than 1.The movement of such securities is slow than market movement, hence called defensive securities. The investors who are risk averse prefer to invest in such securities.

	security Name	mean return(Ri)	var(σ2)	(β)	Rf	market risk (σ2m	unsystmtc risk ei2	excess return(Ri-Rf)	β2	β2σ2m
1	ACC Ltd.	0.0030637	0.0012699	0.8547259	0.0907	0.000539	0.000876	-0.08763632	0.731	0.00039
2	Adani Ports and Special Economic Zone Ltd.	0.0020004	0.0005386	1	0.0907	0.000539	0	-0.08869961	1	0.00054
3	Ambuja Cements Ltd.	0.0039196	0.0016293	0.9735834	0.0907	0.000539	0.001119	-0.08678041	0.948	0.00051
4	Asian Paints Ltd.	0.0066538	0.0012242	0.5780624	0.0907	0.000539	0.001044	-0.0840462	0.334	0.00018
5	Axis Bank Ltd.	0.0167615	0.0615262	0.5832793	0.0907	0.000539	0.061343	-0.07393848	0.34	0.00018
6	Bajaj Auto Ltd.	0.0036226	0.0014004	0.869208	0.0907	0.000539	0.000994	-0.08707741	0.756	0.00041
7	Bank of Baroda	0.0168568	0.0715858	2.1772302	0.0907	0.000539	0.069033	-0.07384315	4.74	0.00255
8	Bharat Heavy Electricals Ltd.	0.0003228	0.0033247	1.3898947	0.0907	0.000539	0.002284	-0.09037723	1.932	0.00104
9	Bharat Petroleum Corporation Ltd.	0.0057794	0.002196	0.9555177	0.0907	0.000539	0.001704	-0.08492062	0.913	0.00049
10	Bharti Airtel Ltd.	0.0019102	0.0016786	0.6813024	0.0907	0.000539	0.001429	-0.08878979	0.464	0.00025
11	Bosch Ltd.	0.0061676	0.0011469	0.4896149	0.0907	0.000539	0.001018	-0.08453239	0.24	0.00013
12	Cairn India Ltd.	-0.00145	0.0011964	0.6714761	0.0907	0.000539	0.000954	-0.09215049	0.451	0.00024
13	Cipla Ltd.	0.0036959	0.0011377	0.4850085	0.0907	0.000539	0.001011	-0.08700405	0.235	0.00013
14	Coal India Ltd.	0.0027205	0.0016178	0.719716	0.0907	0.000539	0.001339	-0.08797945	0.518	0.00028
15	Dr. Reddy's Laboratories Ltd.	0.0049329	0.00071	0.4333541	0.0907	0.000539	0.000609	-0.08576714	0.188	0.0001
16	GAIL (India) Ltd.	0.0002179	0.0011524	0.7563738	0.0907	0.000539	0.000844	-0.09048213	0.572	0.00031
17	Grasim Industries Ltd.	0.0033608	0.0010941	0.9150134	0.0907	0.000539	0.000643	-0.08733915	0.837	0.00045
18	HCL TechLtd.	0.0091868	0.0059404	0.6900569	0.0907	0.000539	0.005684	-0.08151317	0.476	0.00026
19	HDFC Bank Ltd.	0.0045213	0.0009948	1	0.0907	0.000539	0.000456	-0.08617871	1	0.00054
20	Hero MotoCorp Ltd.	0.0028315	0.0013897	0.560089	0.0907	0.000539	0.001221	-0.08786848	0.314	0.00017
21	Hindalco Industries Ltd.	-0.000138	0.0027978	1.4828654	0.0907	0.000539	0.001613	-0.09083824	2.199	0.00118
22	Hindustan Unilever Ltd.	0.002362	0.000539	1	0.0907	0.000539	4.07E-07	-0.088338	1	0.00054
23	Housing Development Finance Corporation Ltd.	0.0039419	0.0013291	1.1310302	0.0907	0.000539	0.00064	-0.08675811	1.279	0.00069
24	I T C Ltd.	0.00359	0.000972	0.61561	0.0907	0.000539	0.000768	-0.08711	0.379	0.0002
25	ICICI Bank Ltd.	0.0171602	0.0700355	1.1953656	0.0907	0.000539	0.069266	-0.0735398	1.429	0.00077
26	Idea Cellular Ltd.	0.0044254	0.002031	0.4605784	0.0907	0.000539	0.001917	-0.0862746	0.212	0.00011
27	IndusInd Bank Ltd.	0.007133	0.002046	1.272	0.0907	0.000539	0.001175	-0.083567	1.618	0.00087

28	Infosys Ltd.	0.0073189	0.0107942	0.8349831	0.0907	0.000539	0.010419	-0.08338106	0.697	0.00038
29	Kotak Mahindra Bank Ltd.	0.0073203	0.0060531	0.9460656	0.0907	0.000539	0.005571	-0.08337971	0.895	0.00048
30	Larsen & Toubro Ltd.	0.0027883	0.0019939	1.412967	0.0907	0.000539	0.000919	-0.08791168	1.996	0.00108
31	Lupin Ltd.	0.0070058	0.0010555	0.1654595	0.0907	0.000539	0.001041	-0.08369416	0.027	1.5E-05
32	Mahindra & Mahindra Ltd.	0.0058484	0.0012507	0.4139615	0.0907	0.000539	0.001158	-0.08485163	0.171	9.2E-05
33	Maruti Suzuki India Ltd.	0.0057499	0.0015454	0.9089712	0.0907	0.000539	0.0011	-0.08495012	0.826	0.00045
34	NTPC Ltd.	-0.000267	0.0014011	0.8299248	0.0907	0.000539	0.00103	-0.09096749	0.689	0.00037
35	Oil & Natural Gas Corporation Ltd.	0.0010588	0.0017331	0.9508059	0.0907	0.000539	0.001246	-0.08964117	0.904	0.00049
36	Power Grid Corporation of India Ltd.	0.0021902	0.000869	0.70355	0.0907	0.000539	0.000602	-0.08850983	0.495	0.00027
37	Punjab National Bank	0.0148775	0.0702462	1.2132483	0.0907	0.000539	0.069453	-0.07582251	1.472	0.00079
38	Reliance Industries Ltd.	0.0008524	0.0013031	1.0884659	0.0907	0.000539	0.000665	-0.08984763	1.185	0.00064
39	State Bank of India	0.043099	0.3869939	1.759977	0.0907	0.000539	0.386994	-0.047601	3.098	
40	Sun Pharmaceutical Industries Ltd.	0.0070733	0.0014452	0.6422134	0.0907	0.000539	0.001223	-0.08362669	0.412	0.00022
41	TCS Ltd	0.004986	0.0011959	0.5896453	0.0907	0.000539	0.001009	-0.08571398	0.348	0.00019
42	Tata Motors Ltd.	0.0050983	0.0025831	1.3946648	0.0907	0.000539	0.001535	-0.08560175	1.945	0.00105
43	Tata Power Co. Ltd.	-0.000601	0.0016168	1.0149435	0.0907	0.000539	0.001062	-0.09130124	1.03	0.00055
44	Tata Steel Ltd.	-0.001265	0.0024281	1.2840231	0.0907	0.000539	0.00154	-0.09196469	1.649	0.00089
45	Tech Mahindra Ltd.	0.0074722	0.0063067	0.6897574	0.0907	0.000539	0.00605	-0.08322784	0.476	0.00026
46	UltraTech Cement Ltd.	0.0057278	0.0014271	0.8300834	0.0907	0.000539	0.001056	-0.08497224	0.689	0.00037
47	Vedanta Ltd.	-0.002196	0.0029729	1.2762112	0.0907	0.000539	0.002096	-0.09289646	1.629	0.00088
48	Wipro Ltd.	0.0020943	0.0013488	0.5358836	0.0907	0.000539	0.001194	-0.0886057	0.287	0.00015
49	Yes Bank Ltd.	0.0058474	0.0033917	0.5358836	0.0907	0.000539	0.003237	-0.08485261	0.287	0.00015
50	Zee Entertainment Enterprises Ltd.	0.0046382	0.0014658	0.5770934	0.0907	0.000539	0.001286	-0.08606177	0.333	0.00018

As proposed by Sharpe, stocks which have negative returns should be ignored for selection in optimal portfolio. The securities on the basis of their returns are ranked from (highest to lowest) for selection. Fischer and Jordan (1995) state that stocks to be included in optimal portfolio are determined on the basis of their ' excess return to beta ratio.' As per the rule of ranking, security having highest 'excess return to beta ratio'will be placed in the first position, followed by the security with second highest

beta ratio, and so on and so forth. It is seen from Table 2 that State Bank of India occupies the first place as per ranking, followed by Axis Bank. Stocks with negative returns are excluded. Out of 50 stocks considered, only 39 stocks are finally selected for consideration.

	security Name	**annual ret**	**var(σ2)**	**(β)**	**market risk σ2m**	**β2σ2m**	**Residual var (σ^2ei)**	**Rf**	**excess re (Ri-Rf)**	**(Ri-Rf)/β**	**β/σ^2ei**	**(Ri-rf)*β/σ^2ei**	**∑(Ri-rf)*β/σ^2ei**	**β^2/σ^2ei**	**∑β^2/σ^2ei**	**cutoff rate**
1	State Bank of India	6.093	0.386993912	1.759977	0.000538593	0.001668302	0.39	0.09	6.0027	3.411	4.567506	27	27.4	8.04	8.0387057	0.193186978
2	Axis Bank Ltd.	1.374	0.061526204	0.583279	0.000538593	0.000183237	0.06	0.09	1.2828	2.199	9.508495	12	39.6	5.55	13.584814	0.193186978
3	Lupin Ltd.	0.438	0.001055487	0.165459	0.000538593	1.4745E-05	0	0.09	0.347	2.097	158.9822	55	94.8	26.3	39.889922	0.193186978
4	ICICI Bank Ltd.	1.422	0.070035502	1.195366	0.000538593	0.000769595	0.07	0.09	1.3317	1.114	17.25763	23	118	20.6	60.519102	0.193186978
5	Punjab National Bank	1.155	0.070246229	1.213248	0.000538593	0.000792794	0.07	0.09	1.0646	0.877	17.46851	19	136	21.2	81.712746	0.193186978
6	HCL Technologies Ltd.	0.609	0.005940376	0.690057	0.000538593	0.000256466	0.01	0.09	0.5182	0.751	121.4053	63	199	83.8	165.48932	0.193186978
7	Mahindra & Mahindra Ltd.	0.354	0.00125069	0.413961	0.000538593	9.22955E-05	0	0.09	0.2635	0.637	357.358	94	293	148	313.42176	0.193186978
8	Bank of Baroda	1.385	0.071585843	2.177723	0.000538593	0.002553109	0.07	0.09	1.2944	0.595	31.5391	41	334	68.7	382.08964	0.193186978
9	Bosch Ltd.	0.377	0.001146866	0.489615	0.000538593	0.000129113	0	0.09	0.2861	0.584	481.0743	138	472	236	617.63079	0.193186978
10	Asian Paints Ltd.	0.412	0.001224178	0.578062	0.000538593	0.000179974	0	0.09	0.3211	0.555	553.5915	178	650	320	937.64125	0.193186978
11	Tech Mahindra Ltd.	0.473	0.006306728	0.689757	0.000538593	0.000256244	0.01	0.09	0.382	0.554	114.0004	44	693	78.6	1016.2738	0.193186978
12	Sun Pharmaceutical Industries Ltd.	0.443	0.00144517	0.642213	0.000538593	0.000222136	0	0.09	0.352	0.548	525.0987	185	878	337	1353.4993	0.193186978
13	Dr. Reddy's Laboratories Ltd.	0.292	0.000709986	0.433354	0.000538593	0.000101146	0	0.09	0.2009	0.464	711.7696	143	1021	308	1661.9476	0.193186978
14	Infosys Ltd.	0.461	0.010794206	0.834983	0.000538593	0.000375505	0.01	0.09	0.3704	0.444	80.14273	30	1051	66.9	1728.8654	0.193186978
15	Kotak Mahindra Bank Ltd.	0.461	0.006053066	0.946066	0.000538593	0.000482062	0.01	0.09	0.3705	0.392	169.8196	63	1114	161	1889.5259	0.193186978

16	Idea Cellular Ltd.	0.258	0.00203095	0.460578	0.000538593	0.000114253	0	0.09	0.1674	0.363	240.298	40	1154	111	2000.202	0.193186978
17	Tata Consultancy Services Ltd.	0.295	0.001195945	0.589645	0.000538593	0.000187259	0	0.09	0.2045	0.347	584.5677	120	1273	345	2344.8896	0.193186978
18	Zee Entertainment Enterprises Ltd.	0.272	0.001465841	0.577093	0.000538593	0.000179371	0	0.09	0.1814	0.314	448.5869	81	1355	259	2603.7661	0.193186978
19	UltraTech Cement Ltd.	0.346	0.001427095	0.830083	0.000538593	0.000371111	0	0.09	0.2551	0.307	786.0759	201	1555	653	3256.2747	0.193186978
20	Maruti Suzuki India Ltd.	0.347	0.001545416	0.908971	0.000538593	0.000445001	0	0.09	0.2566	0.282	826.0258	212	1767	751	4007.1083	0.193186978
21	IndusInd Bank Ltd.	0.447	0.002046	1.272	0.000538593	0.000871435	0	0.09	0.3565	0.28	1082.954	386	2153	1378	5384.6257	0.193186978
22	Bharat Petroleum Corporation Ltd.	0.349	0.002195968	0.955518	0.000538593	0.000491743	0	0.09	0.2587	0.271	560.6758	145	2298	536	5920.3614	0.193186978
23	Hindustan Unilever Ltd.	0.35	0.001361389	1	0.000538593	0.000538593	0	0.09	0.2593	0.259	1215.368	315	2613	1215	7135.7295	0.193186978
24	Cipla Ltd.	0.211	0.001137738	0.485008	0.000538593	0.000126695	0	0.09	0.1208	0.249	479.711	58	2671	233	7368.3934	0.193186978

The securities are arranged in descending order on the basis of their 'excess return to beta ratio.' finally the value of C* for each of the securities are calculated using the formula

$$Ci=\frac{\sigma^2m\sum(Ri-Rf)\beta/\sigma^ei}{1+\sigma^2m\sum \beta i^2/\sigma^ei}$$

From the table it is found that excess return to beta ratio of 24 securities exceed the value of C*. The cut off rate is .19. Out of total 50 stocks considered for the study, only 24 stocks having their 'excess return to beta ratio'above cut off rate of (.19) are eligible for inclusion in optimal portfolio. It can be seen from the (Table 2) that in the optimal portfolio maximum stocks are from banks. Banks like SBI, Axis Bank, PNB, Bank of Baroda, ICICI, Kodak Mahindra and IndusInd bank are part of the optimal.

Portfolio. In optimal portfolio from pharmaceutical sector, stocks of Dr Reddy's Lab, Cipla Ltd and Sun Pharmaceuticals are included. The stocks of companies like Zee Entertainment, TCS and Maruti Suzuki Ltd are also a part of this portfolio.

Once the composition of stocks included in optimal portfolio is determined, the final step is to find proportion of each of these selected stocks in portfolio. The proportion of amount to be invested in each stock gives a fair idea to the investor. It describes as to how the total investment needs to be allocated amongst the various selected stocks to get the maximum benefits of the optimum portfolio. Table 3 shows the proportion to be invested in various stocks which comprise of the optimal portfolio.

Sno	security Name	∑β^2/σ^2ei	cutoff rate	Zi	∑Zi	Wi (Prop of investment)	% of investment
1	State Bank of India	8.038706	0.193187	25.86441	25.86441	0.020726	2.07261
2	Axis Bank Ltd.	13.58481	0.193187	11.12627	36.99068	0.008916	0.891589
3	Lupin Ltd.	39.88992	0.193187	50.08246	87.07314	0.040133	4.013291
4	ICICI Bank Ltd.	60.5191	0.193187	18.99677	106.0699	0.015223	1.522281
5	Punjab National Bank	81.71275	0.193187	14.5027	120.5726	0.011622	1.162154
6	HCL Technologies Ltd.	165.4893	0.193187	46.72449	167.2971	0.037442	3.744205
7	Mahindra & Mahindra Ltd.	313.4218	0.193187	65.59415	232.8912	0.052563	5.2563
8	Bank of Baroda	382.0896	0.193187	27.55915	260.4504	0.022084	2.208416
9	Bosch Ltd.	617.6308	0.193187	92.1111	352.5615	0.073812	7.381201
10	Asian Paints Ltd.	937.6413	0.193187	115.9257	468.4872	0.092896	9.289551
11	Tech Mahindra Ltd.	1016.274	0.193187	28.35913	496.8463	0.022725	2.272521
12	Sun Pharmaceutical Industries Ltd.	1353.499	0.193187	119.6879	616.5342	0.09591	9.59103
13	Dr. Reddy's Laboratories Ltd.	1661.948	0.193187	83.40288	699.9371	0.066834	6.683379
14	Infosys Ltd.	1728.865	0.193187	16.75834	716.6954	0.013429	1.342907
15	Kotak Mahindra Bank Ltd.	1889.526	0.193187	31.88328	748.5787	0.025549	2.554925
16	Idea Cellular Ltd.	2000.202	0.193187	18.84782	767.4265	0.015103	1.510345
17	Tata Consultancy Services Ltd.	2344.89	0.193187	52.92756	820.3541	0.042413	4.24128
18	Zee Entertainment Enterprises Ltd.	2603.766	0.193187	31.3401	851.6942	0.025114	2.511397
19	UltraTech Cement Ltd.	3256.275	0.193187	74.47742	926.1716	0.059681	5.968149
20	Maruti Suzuki India Ltd.	4007.108	0.193187	66.94579	993.1174	0.053646	5.364612
21	IndusInd Bank Ltd.	5384.626	0.193187	119.9057	1113.023	0.096085	9.60848
22	Bharat Petroleum	5920.361	0.193187	41.55157	1154.575	0.033297	3.32968

	Corporation Ltd.						
23	Hindustan Unilever Ltd.	7135.73	0.193187	80.35294	1234.928	0.06439	6.438976
24	Cipla Ltd.	7368.393	0.193187	12.98733	1247.915	0.010407	1.040722

The highest proportion of investment of 9.60 percent is to be made in IndusInd Bank and Sun Pharrmaceuticals, followed by 9.28 percent in Asian Paints. The table shows that 7.38 percent has to be invested in Bosch Ltd and 1.5 percent in ICICI bank. The lowest proportion of 0.89 percent has to be invested in Axis Bank.

Findings

1. It is found that out of 50 stocks considered for study, only 24 stocks are chosen for inclusion in optimal portfolio. The 'excess return to beta ratio' of only 24 stocks was above the calculated cutoff rate of .19.
2. Out of 24 stocks selected, the maximum number of stocks is from the banking sector. Stocks of SBI, PNB, IndusInd bank, ICICI bank and Axis Bank are a part of optimal portfolio. Stocks of companies like Dr Reddy's Lab, Cipla and Sun Pharmaceuticals from the pharmaceutical sector are also included in the optimal portfolio.
3. It was found in the study that maximum proportion of 9.6 percent should be invested in IndusInd Bank and Sun Pharmaceuticals and least .89 percent is to be invested in Axis Bank.
4. Systematic risk is less than Unsystematic risk in SIM .It can be reduced through diversification.
5. In the study it was found beta of most of the banks was more than with Bank of Baroda having beta of more than 2. These stocks are said to be aggressive as their returns change more than proportional change in market.
6. Majority of the stocks are found to have their beta less than 1 and hence can be termed as defensive securities. Investors who are risk averse may prefer to invest in such type of securities.

Implications of the Study

With the significant increase in stock market operations post economic reforms, this model can be used by investors who want to create a portfolio and invest in stocks to diversify risk. In recent years stocks market has attracted huge number of potential and prospective investors who lack professional and practical experience to invest in stock market. This model can be of great help to both present and prospective investors who wish to earn consistent returns with safety. The study covers a period of around five years and can give useful insights regarding the performance of securities to the prospective investors. By providing information about the beta, variances, systematic and unsystematic risk of the securities, this study gives a fair idea about existing risk and volatility of the chosen stocks. Stocks need to fulfill stringent selection criteria to be a part of Nifty 50 regarding profitability, performance and safety of investors. As the present study constructs optimum portfolio using stocks of Nifty 50, the investors can be assured of returns and safety. It also provides information on the proportion of investment to be made in each selected stock of the optimal portfolio. The results of this study can be used both by individual and institutional investors in creating optimal portfolio to diversify and reduce risk and enhance their returns.

Limitations and Directions for Future Research

Although the study holds good in Indian context, it suffers from certain limitations.

The major limitation with SIM is that it holds true only for a single point of time. The dynamism and uncertainty prevalent in the markets with the passage of time is not taken into account. This model assumes that co-movement of security prices is only due to the common market movement together only with the market. (Mandal, 2008).There are conditions beyond the general market and business conditions like the industry oriented factors which cause the securities prices to move together. (Gregory and Shapiro, 2001 and Campbell et.al, 2001). As per (Chandra, 2009) empirical evidences have shown that complicated models have not been able to outperform the ability of single index model to predict the covariances existing between security return.

For this study, the stocks from Nifty 50 are taken into consideration. Stocks from other such similar indices can be used to create other diversified portfolios. Studies can be undertaken to compare the performances of two or more indices and decisions can be taken based on their returns and total risk. Stock market indices of two or more countries can be combined to create efficient and diversified portfolios for investors who are desirous to invest in global stock markets. Further research can be can be carried out by combining several portfolio models such as Markowitz Model, Sharpe's Single Index Model and Capital Asset Pricing Model for creating better and efficient optimal portfolios.

Conclusion

The task of construction of optimal portfolio is tough and challenging both for institutional as well as individual investors. This paper attempts to construct an optimal portfolio taking 50 stocks of Nifty 50 Index. As evident from the above study, only 24 stocks fulfill the

selection criteria of being included in Optimal portfolio.Majority of the stocks who are part of the optimal portfolio are from the banking sector.This indicates that financial sector is growing rapidly and stocks of financial sector are providing consistent and assured returns.There are macro and general economic factors which also affect the securities movement and their selection. These factors should also be considered while selecting securities for optimal portfolio (Nalini, 2014). In emerging economies like India where Foreign Institutional Investors are investing through stock markets, the role of this model has increased manifold (Mandal, 2013; and Nalini, 2014).In recent years the rising Sensex and Nifty have also attracted potential investors towards stock market for consistent returns and assured returns. It has become a lucrative avenue for investment. This study can be helpful in the field of investment finance. Further studies can be made on other indices and a variety of optimal portfolio can be created.

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